■ Déterminer les éventuelles asymptotes des fonctions suivantes

1. $f(x) = \sqrt{x-2} - \sqrt{x+3}$

2. $f(x) = \sqrt{4x^2 - 4x - 8} - 2x$

3. $f(x) = \dfrac{2x^2 + 5x + 2}{x+3}$

4. $f(x) = \dfrac{1 - 2x}{2x^2 + x - 1}$

5. $f(x) = \dfrac{x^3 - x}{x^2 + 2x + 1}$

6. $f(x) = \dfrac{2x^2 - 5x - 3}{3 - x}$

7. $f(x) = \dfrac{2x + 3}{x^2 + 3x + 2}$

8. $f(x) = \dfrac{3x^2 + 2x - 1}{x^2 - x - 2}$

9. $f(x) = \dfrac{x^2 + 2x - 3}{2x^2 + 3x - 9}$

10. $f(x) = \dfrac{x^2 + x - 2}{x - x^3}$

11. $f(x) = \dfrac{2x^2 - 3x - 2}{2 - x}$

12. $f(x) = \sqrt{x^2 + x - 12} - \sqrt{x^2 - 3x - 10}$

13. $f(x) = \dfrac{\sqrt{3x+1} - \sqrt{2x+6}}{x - 5}$

■ Solutions

1. Dom f = [2, ⟶

$\lim_{x\to 2} \sqrt{x-2} - \sqrt{x+3} = -\sqrt{5}$

$\lim_{x\to +\infty} \sqrt{x-2} - \sqrt{x+3} = 0$

$\lim_{x\to -\infty} \sqrt{x-2} - \sqrt{x+3}$ n'existe pas

AH ≡ $y = 0$ à droite

2. Dom f = ⟵, −1] ∪ [2, ⟶

$\lim_{x\to -1} \sqrt{4x^2 - 4x - 8} - 2x = 2$

$\lim_{x\to 2} \sqrt{4x^2 - 4x - 8} - 2x = -4$

$\lim_{x\to +\infty} \sqrt{4x^2 - 4x - 8} - 2x = -1$

$\lim_{x\to -\infty} \sqrt{4x^2 - 4x - 8} - 2x = +\infty$

AH ≡ $y = -1$ à droite

3. Dom f = ℝ \ {−3}

$$\begin{cases} \lim\limits_{\substack{x\to -3 \\ <}} \frac{2x^2+5x+2}{x+3} = -\infty \\ \lim\limits_{\substack{x\to -3 \\ >}} \frac{2x^2+5x+2}{x+3} = +\infty \end{cases}$$

AV ≡ $x = -3$

$$\lim_{x\to +\infty} \frac{2x^2+5x+2}{x+3} = +\infty$$

$$\lim_{x\to -\infty} \frac{2x^2+5x+2}{x+3} = -\infty$$

AO ≡ $y = 2x - 1$

4. Dom f = ℝ \ $\{-1, \frac{1}{2}\}$

$$\begin{cases} \lim\limits_{\substack{x\to -1 \\ <}} \frac{1-2x}{2x^2+x-1} = +\infty \\ \lim\limits_{\substack{x\to -1 \\ >}} \frac{1-2x}{2x^2+x-1} = -\infty \end{cases}$$

AV ≡ $x = -1$

$$\lim_{x\to \frac{1}{2}} \frac{1-2x}{2x^2+x-1} = -\frac{2}{3}$$

$$\lim_{x\to +\infty} \frac{1-2x}{2x^2+x-1} = 0$$

$$\lim_{x\to -\infty} \frac{1-2x}{2x^2+x-1} = 0$$

AH ≡ $y = 0$

5. Dom f = ℝ \ {−1}

$$\begin{cases} \lim\limits_{\substack{x\to -1 \\ <}} \frac{x^3-x}{x^2+2x+1} = -\infty \\ \lim\limits_{\substack{x\to -1 \\ >}} \frac{x^3-x}{x^2+2x+1} = +\infty \end{cases}$$

AV ≡ $x = -1$

$$\lim_{x\to +\infty} \frac{x^3-x}{x^2+2x+1} = +\infty$$

$$\lim_{x\to -\infty} \frac{x^3-x}{x^2+2x+1} = -\infty$$

AO ≡ $y = x - 2$

6. Dom f = ℝ \ {3}

$$\lim_{x\to 3} \frac{2x^2-5x-3}{3-x} = -7$$

$$\lim_{x\to +\infty} \frac{2x^2-5x-3}{3-x} = -\infty$$

$$\lim_{x\to -\infty} \frac{2x^2-5x-3}{3-x} = +\infty$$

AO ≡ $y = -2x - 1$

7. Dom f = ℝ \ {−2,−1}

$$\begin{cases} \lim\limits_{\substack{x\to -2\\ <}} \frac{2x+3}{x^2+3x+2} = -\infty \\ \lim\limits_{\substack{x\to -2\\ >}} \frac{2x+3}{x^2+3x+2} = +\infty \end{cases}$$

AV $\equiv x = -2$

$$\begin{cases} \lim\limits_{\substack{x\to -1\\ <}} \frac{2x+3}{x^2+3x+2} = -\infty \\ \lim\limits_{\substack{x\to -1\\ >}} \frac{2x+3}{x^2+3x+2} = +\infty \end{cases}$$

AV $\equiv x = -1$

$$\lim_{x\to +\infty} \frac{2x+3}{x^2+3x+2} = 0$$

$$\lim_{x\to -\infty} \frac{2x+3}{x^2+3x+2} = 0$$

AH $\equiv y = 0$

8. Dom f = $\mathbb{R} \setminus \{-1,2\}$

$$\lim_{x\to -1} \frac{3x^2+2x-1}{x^2-x-2} = \frac{4}{3}$$

$$\begin{cases} \lim\limits_{\substack{x\to 2\\ <}} \frac{3x^2+2x-1}{x^2-x-2} = -\infty \\ \lim\limits_{\substack{x\to 2\\ >}} \frac{3x^2+2x-1}{x^2-x-2} = +\infty \end{cases}$$

AV $\equiv x = 2$

$$\lim_{x\to +\infty} \frac{3x^2+2x-1}{x^2-x-2} = 3$$

$$\lim_{x\to -\infty} \frac{3x^2+2x-1}{x^2-x-2} = 3$$

AH $\equiv y = 3$

9. Dom f = $\mathbb{R} \setminus \{-3, \frac{3}{2}\}$

$$\lim_{x\to -3} \frac{x^2+2x-3}{2x^2+3x-9} = \frac{4}{9}$$

$$\begin{cases} \lim\limits_{\substack{x\to \frac{3}{2}\\ <}} \frac{x^2+2x-3}{2x^2+3x-9} = -\infty \\ \lim\limits_{\substack{x\to \frac{3}{2}\\ >}} \frac{x^2+2x-3}{2x^2+3x-9} = +\infty \end{cases}$$

AV $\equiv x = \frac{3}{2}$

$$\lim_{x\to +\infty} \frac{x^2+2x-3}{2x^2+3x-9} = \frac{1}{2}$$

$$\lim_{x\to -\infty} \frac{x^2+2x-3}{2x^2+3x-9} = \frac{1}{2}$$

AH $\equiv y = \frac{1}{2}$

10. Dom f = $\mathbb{R} \setminus \{-1,0,1\}$

$$\begin{cases} \lim\limits_{\substack{x\to -1\\ <}} \frac{x^2+x-2}{x-x^3} = -\infty \\ \lim\limits_{\substack{x\to -1\\ >}} \frac{x^2+x-2}{x-x^3} = +\infty \end{cases}$$

AV ≡ $x = -1$

$$\begin{cases} \lim\limits_{\substack{x\to 0\\ <}} \frac{x^2+x-2}{x-x^3} = +\infty \\ \lim\limits_{\substack{x\to 0\\ >}} \frac{x^2+x-2}{x-x^3} = -\infty \end{cases}$$

AV ≡ $x = 0$

$$\lim_{x\to 1} \frac{x^2+x-2}{x-x^3} = -\frac{3}{2}$$

$$\lim_{x\to +\infty} \frac{x^2+x-2}{x-x^3} = 0$$

$$\lim_{x\to -\infty} \frac{x^2+x-2}{x-x^3} = 0$$

AH ≡ $y = 0$

11. Dom f = ℝ \ {2}

$$\lim_{x\to 2} \frac{2x^2-3x-2}{2-x} = -5$$

$$\lim_{x\to +\infty} \frac{2x^2-3x-2}{2-x} = -\infty$$

$$\lim_{x\to -\infty} \frac{2x^2-3x-2}{2-x} = +\infty$$

AO ≡ $y = -2x - 1$

12. Dom f = ⟵, −4] ∪ [5, ⟶

$$\lim_{x\to -4} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = -3\sqrt{2}$$

$$\lim_{x\to 5} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = 3\sqrt{2}$$

$$\lim_{x\to +\infty} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = 2$$

$$\lim_{x\to -\infty} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = -2$$

AH ≡ $y = 2$ à droite

AH ≡ $y = -2$ à gauche

13. Dom f = $[-\frac{1}{3}, 5[\cup]5, \longrightarrow$

$$\lim_{x\to -\frac{1}{3}} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} = \frac{\sqrt{3}}{4}$$

$$\lim_{x\to 5} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} = \frac{1}{8}$$

$$\lim_{x\to +\infty} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} = 0$$

$$\lim_{x\to -\infty} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} \quad \text{n'existe pas}$$

AH ≡ $y = 0$ à droite