Calcul de limites

- Indéterminations $\left[\dfrac{k}{0}\right]$ et $\left[\dfrac{0}{0}\right]$

1) $\lim\limits_{x\to-1} \dfrac{x^2 - x - 2}{x^2 - 2x - 3}$

2) $\lim\limits_{x\to 2} \dfrac{x^2 - 4x + 4}{x^2 - 4}$

3) $\lim\limits_{x\to-3} \dfrac{x^2 + 3x}{x^2 + 6x + 9}$

4) $\lim\limits_{x\to 1} \dfrac{x^3 - 1}{x - 1}$

5) $\lim\limits_{x\to 2} \dfrac{x^2 - 4}{x^3 - 8}$

6) $\lim\limits_{x\to\frac{1}{2}} \dfrac{4x^2 - 4x + 1}{2x^2 - x}$

7) $\lim\limits_{x\to-4} \dfrac{2x^2 + 9x + 4}{3x^2 + 11x - 4}$

8) $\lim\limits_{x\to-1} \dfrac{2x^3 - 5x^2 - 8x - 1}{x^4 + x^3 - x - 1}$

9) $\lim\limits_{x\to 1} \dfrac{x^3 - x^2 - x + 1}{x^2 - 3x + 2}$

10) $\lim\limits_{x\to 1} \dfrac{x^3 + x^2 - 5x + 3}{x^3 - 3x^2 + 3x - 1}$

- Solutions

1)

$$\lim\limits_{x\to-1} \dfrac{x^2 - x - 2}{x^2 - 2x - 3} = \left[\dfrac{0}{0}\right]$$

$$= \lim\limits_{x\to-1} \dfrac{(x - 2)(x + 1)}{(x - 3)(x + 1)}$$

$$= \lim\limits_{x\to-1} \dfrac{x - 2}{x - 3}$$

$$\lim\limits_{x\to-1} \dfrac{x^2 - x - 2}{x^2 - 2x - 3} = \dfrac{3}{4}$$

2)

$$\lim\limits_{x\to 2} \dfrac{x^2 - 4x + 4}{x^2 - 4} = \left[\dfrac{0}{0}\right]$$

$$= \lim\limits_{x\to 2} \dfrac{(x - 2)^2}{(x - 2)(x + 2)}$$

$$= \lim\limits_{x\to 2} \dfrac{x - 2}{x + 2}$$

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 - 4} = 0$$

3)

$$\lim_{x \to -3} \frac{x^2 + 3x}{x^2 + 6x + 9} = [\frac{0}{0}]$$

$$= \lim_{x \to -3} \frac{x(x + 3)}{(x + 3)^2}$$

$$= \lim_{x \to -3} \frac{x}{x + 3}$$

x		-3		0	
$\frac{x^2+3x}{x^2+6x+9}$	$+$	$\vert$	$-$	0	$+$

$$\begin{cases} \lim_{\substack{x \to -3 \\ <}} \frac{x^2+3x}{x^2+6x+9} = +\infty \\ \lim_{\substack{x \to -3 \\ >}} \frac{x^2+3x}{x^2+6x+9} = -\infty \end{cases}$$

4)

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = [\frac{0}{0}]$$

$$= \lim_{x \to 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$$

$$= \lim_{x \to 1} x^2 + x + 1$$

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3$$

5)

$$\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8} = [\frac{0}{0}]$$

$$= \lim_{x \to 2} \frac{(x - 2)(x + 2)}{(x - 2)(x^2 + 2x + 4)}$$

$$= \lim_{x \to 2} \frac{x + 2}{x^2 + 2x + 4}$$

$$\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8} = \frac{1}{3}$$

6)

$$\lim_{x \to \frac{1}{2}} \frac{4x^2 - 4x + 1}{2x^2 - x} = [\frac{0}{0}]$$

$$= \lim_{x \to \frac{1}{2}} \frac{(2x - 1)^2}{x(2x - 1)}$$

$$= \lim_{x \to \frac{1}{2}} \frac{2x - 1}{x}$$

$$\lim_{x \to \frac{1}{2}} \frac{4x^2 - 4x + 1}{2x^2 - x} = 0$$

7)

$$\lim_{x \to -4} \frac{2x^2 + 9x + 4}{3x^2 + 11x - 4} = [\frac{0}{0}]$$

$$= \lim_{x \to -4} \frac{(x+4)(2x+1)}{(x+4)(3x-1)}$$

$$= \lim_{x \to -4} \frac{2x+1}{3x-1}$$

$$\lim_{x \to -4} \frac{2x^2+9x+4}{3x^2+11x-4} = \frac{7}{13}$$

8)

$$\lim_{x \to -1} \frac{2x^3-5x^2-8x-1}{x^4+x^3-x-1} = [\frac{0}{0}]$$

$$= \lim_{x \to -1} \frac{(x+1)(2x^2-7x-1)}{(x-1)(x+1)(x^2+x+1)}$$

$$= \lim_{x \to -1} \frac{2x^2-7x-1}{(x-1)(x^2+x+1)}$$

$$\lim_{x \to -1} \frac{2x^3-5x^2-8x-1}{x^4+x^3-x-1} = -4$$

9)

$$\lim_{x \to 1} \frac{x^3-x^2-x+1}{x^2-3x+2} = [\frac{0}{0}]$$

$$= \lim_{x \to 1} \frac{(x-1)^2(x+1)}{(x-2)(x-1)}$$

$$= \lim_{x \to 1} \frac{(x-1)(x+1)}{x-2}$$

$$\lim_{x \to 1} \frac{x^3-x^2-x+1}{x^2-3x+2} = 0$$

10)

$$\lim_{x \to 1} \frac{x^3+x^2-5x+3}{x^3-3x^2+3x-1} = [\frac{0}{0}]$$

$$= \lim_{x \to 1} \frac{(x-1)^2(x+3)}{(x-1)^3}$$

$$= \lim_{x \to 1} \frac{x+3}{x-1}$$

x		-3		1	
$\frac{x^3+x^2-5x+3}{x^3-3x^2+3x-1}$	$+$	0	$-$	$\vert$	$+$

$$\begin{cases} \lim_{\substack{x \to 1 \\ <}} \frac{x^3+x^2-5x+3}{x^3-3x^2+3x-1} = -\infty \\ \lim_{\substack{x \to 1 \\ >}} \frac{x^3+x^2-5x+3}{x^3-3x^2+3x-1} = +\infty \end{cases}$$