

■ Etude de fonction

$$f(x) = \frac{x^2}{(x+1)^3}$$

1. Domaine de définition

$$\text{Dom } f = \mathbb{R} \setminus \{-1\}$$

| | | | |
|-----------------------|--|----|---|
| x | | -1 | 0 |
| $\frac{x^2}{(x+1)^3}$ | | - | + |

2. Limites et asymptotes

$$\lim_{x \rightarrow -1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = \infty$$

$$\text{AV} \equiv x = -1$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

$$\text{AH} \equiv y = 0 \text{ à droite}$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$\text{AH} \equiv y = 0 \text{ à gauche}$$

3. Etude de f'

$$f'(x) = \frac{2x-x^2}{(x+1)^4}$$

| | | | | |
|--------------------------|--|----|---|---|
| x | | -1 | 0 | 2 |
| $\frac{2x-x^2}{(x+1)^4}$ | | - | + | - |

$$\text{Min} = (0, 0)$$

$$\text{Max} = (2, \frac{4}{27})$$

4. Etude de f''

$$f''(x) = \frac{2(x^2 - 4x + 1)}{(x+1)^5}$$

| | | | | |
|-----------------------------------|--|----|----------------|----------------|
| x | | -1 | $2 - \sqrt{3}$ | $2 + \sqrt{3}$ |
| $\frac{2(x^2 - 4x + 1)}{(x+1)^5}$ | | - | + | - |

$$I = (0.267949, 0.0352208)$$

$$I = (3.73205, 0.131446)$$

5. Graph de f

