- **Quotient de polynômes**

  - **Exemple 1**

$$\int \frac{2x^2+x+1}{x^2-3x+4}\,dx$$

$$= \int \frac{2x^2-6x+8+7x-7}{x^2-3x+4}\,dx$$

$$= \int \left(2+\frac{7x-7}{x^2-3x+4}\right)dx$$

$$= 2x+\frac{7}{2}\int \frac{2x-3+1}{x^2-3x+4}\,dx$$

$$= 2x+\frac{7}{2}\int \frac{2x-3}{x^2-3x+4}\,dx+\frac{7}{2}\int \frac{1}{x^2-3x+4}\,dx$$

$$= 2x+\frac{7}{2}\ln\left|x^2-3x+4\right|+\frac{7}{2}\int \frac{1}{x^2-3x+\frac{9}{4}+\frac{7}{4}}\,dx$$

$$= 2x+\frac{7}{2}\ln\left|x^2-3x+4\right|+\frac{7}{2}\int \frac{1}{\left(x-\frac{3}{2}\right)^2+\frac{7}{4}}\,dx$$

$$= 2x+\frac{7}{2}\ln\left|x^2-3x+4\right|+\frac{7}{2}.\frac{4}{7}\int \frac{1}{\frac{\left(x-\frac{3}{2}\right)^2}{\frac{7}{4}}+1}\,dx$$

$$= 2x+\frac{7}{2}\ln\left|x^2-3x+4\right|+2\int \frac{1}{\left(\frac{2x-3}{\sqrt{7}}\right)^2+1}\,dx$$

$$= 2x+\frac{7}{2}\ln\left|x^2-3x+4\right|+2.\frac{\sqrt{7}}{2}\int \frac{\frac{2}{\sqrt{7}}}{\left(\frac{2x-3}{\sqrt{7}}\right)^2+1}\,dx$$

$$= 2x+\frac{7}{2}\ln\left|x^2-3x+4\right|+\sqrt{7}\ \mathrm{Arctg}\left(\frac{2x-3}{\sqrt{7}}\right)+k$$

Exemple 2

$$\int \frac{x^2+2x-5}{x^2-3x+2}\,dx$$

$$= \int \frac{x^2-3x+2+5x-7}{x^2-3x+2}\,dx$$

$$= \int\left(1+\frac{5x-7}{x^2-3x+2}\right)dx$$

$$= x+\frac{1}{2}\int \frac{10x-15+1}{x^2-3x+2}\,dx$$

$$= x+\frac{1}{2}\int \frac{10x-15}{x^2-3x+2}\,dx+\frac{1}{2}\int \frac{1}{x^2-3x+2}\,dx$$

$$= x+\frac{5}{2}\ln\left|x^2-3x+2\right|+\frac{1}{2}\int \frac{1}{(x-1)(x-2)}\,dx$$

$$= x+\frac{5}{2}\ln\left|x^2-3x+2\right|+\frac{1}{2}\int\left(\frac{1}{x-2}-\frac{1}{x-1}\right)dx$$

$$= x+\frac{5}{2}\ln\left|x^2-3x+2\right|+\frac{1}{2}\ln\left|\frac{x-2}{x-1}\right|+k$$

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$$= \int\left(1+\frac{5x-7}{x^2-3x+2}\right)dx$$

$$= x+\int \frac{3x-3+2x-4}{(x-2)(x-1)}\,dx$$

$$= x+\int\left(\frac{3}{x-2}+\frac{2}{x-1}\right)dx$$

$$= x+3\ln\left|x-2\right|+2\ln\left|x-1\right|+k$$

Exemple 3

$$\int \frac{x^2+5x-2}{2x-3}\,dx$$

$$= \frac{1}{2}\int \frac{2x^2-3x+13x-4}{2x-3}\,dx$$

$$= \frac{1}{2}\int\left(x+\frac{13x-4}{2x-3}\right)dx$$

$$= \frac{x^2}{4}+\frac{1}{2}\int \frac{13x-4}{2x-3}\,dx$$

$$= \frac{x^2}{4}+\frac{1}{4}\int \frac{26x-39+31}{2x-3}\,dx$$

$$= \frac{x^2}{4} + \frac{13}{4}\int \frac{2x-3}{2x-3}\,dx + \frac{1}{4}\int \frac{31}{2x-3}\,dx$$

$$= \frac{x^2}{4} + \frac{13x}{4} + \frac{1}{4}\int \frac{31}{2x-3}\,dx$$

$$= \frac{x^2}{4} + \frac{13x}{4} + \frac{31}{8}\int \frac{2}{2x-3}\,dx$$

$$= \frac{x^2}{4} + \frac{13x}{4} + \frac{31}{8}\ln\left|2x-3\right| + k$$

■ Exercices

Calculer

$$\int \frac{x+3}{x^2+6}\,dx$$

$$\int \frac{x+1}{x^2-x-6}\,dx$$

$$\int \frac{2x-1}{x^2+3}\,dx$$

$$\int \frac{2x^2+x-1}{x+2}\,dx$$

$$\int \frac{x-3}{x^2+2x}\,dx$$

Solutions

$$\int \frac{x+3}{x^2+6}\,dx = \sqrt{\frac{3}{2}}\ \text{Arctg}\left(\frac{x}{\sqrt{6}}\right) + \frac{1}{2}\ln\left(\left|x^2+6\right|\right) + \text{k}$$

$$\int \frac{x+1}{x^2-x-6}\,dx = \frac{4}{5}\ln\left(\left|\frac{x-3}{x+2}\right|\right) + \ln(|x+2|) + \text{k}$$

$$\int \frac{2x-1}{x^2+3}\,dx = \ln\left(\left|x^2+3\right|\right) - \frac{\text{Arctg}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} + \text{k}$$

$$\int \frac{2x^2+x-1}{x+2}\,dx = (x+2)^2 - 7(x+2) + 5\ln(|x+2|) + \text{k}$$

$$\int \frac{x-3}{x^2+2x}\,dx = \ln(|x+2|) - \frac{3}{2}\ln\left(\left|\frac{x}{x+2}\right|\right) + \text{k}$$