DÉTERMINER LE DOMAINE DE DÉFINITION DES FONCTIONS SUIVANTES:

- 1) $f(x) = \text{Arcsin}\left(3 - x^2\right)$
- 2) $f(x) = \text{Arctg}\left(\frac{1}{x - 2}\right)$
- 3) $f(x) = \text{Arccos}\left(x^2 - 2x\right)$
- 4) $f(x) = \frac{2}{\text{Arcsin}(3x)}$
- 5) $f(x) = \frac{\text{Arccos}(2x)}{\text{Arctg}\left(\frac{x}{2}\right)}$
- 6) $f(x) = \text{Arcsin}\left(-x^2 - 2x + 1\right)$
- 7) $f(x) = \text{Arcsin}(2 - 5x)$
- 8) $f(x) = \text{Arccos}\left(\frac{1}{x}\right)$
- 9) $f(x) = \text{Arcsin}(3x) - \text{Arccos}(2x)$
- 10) $f(x) = \text{Arctg}\left(6x^2 - 3x\right)$
- 11) $f(x) = \text{Arccos}\left(\frac{x}{x + 1}\right)$
- 12) $f(x) = \text{Arctg}\,\frac{2x + 1}{x + 1}$

Solutions détaillées

- 1) $f(x) = \text{Arcsin}(3 - x^2)$

$-1 \leq 3 - x^2 \leq 1$

$-1 \leq 3 - x^2$ et $3 - x^2 \leq 1$

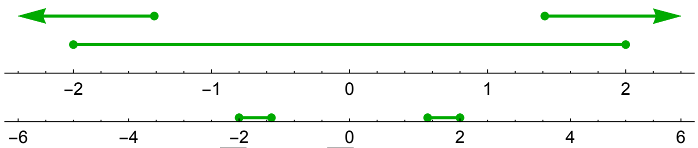
$x^2 - 4 \leq 0$ et $2 - x^2 \leq 0$

$x^2 - 4 \leq 0$ et $2 - x^2 \leq 0$

x		-2		2	
$x^2 - 4$	+	0	−	0	+

x		$-\sqrt{2}$		$\sqrt{2}$	
$2 - x^2$	−	0	+	0	−

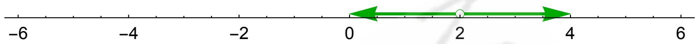
$([-2; 2]) \cap (\longleftarrow; -\sqrt{2}\,] \cup [\sqrt{2}\,; \longrightarrow) = [-2; -\sqrt{2}\,] \cup [\sqrt{2}\,; 2]$



dom $f = [-2; -\sqrt{2}\,] \cup [\sqrt{2}\,; 2]$

- 2) $f(x) = \text{Arctg}\left(\dfrac{1}{x - 2}\right)$

$x - 2 \neq 0 \Leftrightarrow x \neq 2 \Leftrightarrow x \in \mathbb{R} \setminus \{2\}$



dom $f = \mathbb{R} \setminus \{2\}$

- 3) $f(x) = \text{Arccos}(x^2 - 2x)$

$-1 \leq x^2 - 2x \leq 1$

$-1 \leq x^2 - 2x$ et $x^2 - 2x \leq 1$

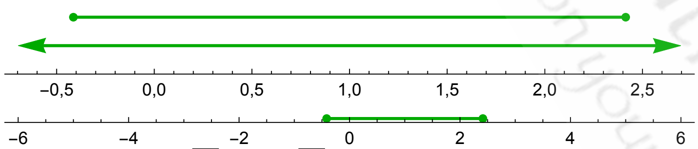
$-x^2 + 2x - 1 \leq 0$ et $x^2 - 2x - 1 \leq 0$

$-x^2 + 2x - 1 \leq 0$ et $x^2 - 2x - 1 \leq 0$

x		1	
$-x^2 + 2x - 1$	−	0	−

x		$1 - \sqrt{2}$		$1 + \sqrt{2}$	
$x^2 - 2x - 1$	+	0	−	0	+

$(\mathbb{R}) \cap ([1 - \sqrt{2}\,; 1 + \sqrt{2}\,]) = [1 - \sqrt{2}\,; 1 + \sqrt{2}\,]$



dom $f = [1 - \sqrt{2}\,; 1 + \sqrt{2}\,]$

- 4) $f(x) = \dfrac{2}{\text{Arcsin}(3x)}$

$\text{Arcsin}(3x) \neq 0 \iff x \neq 0$

et $-1 \leq 3x \leq 1$

$-1 \leq 3x$ et $3x \leq 1$

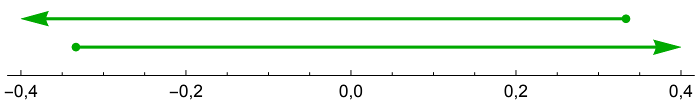
$-3x - 1 \leq 0$ et $3x - 1 \leq 0$

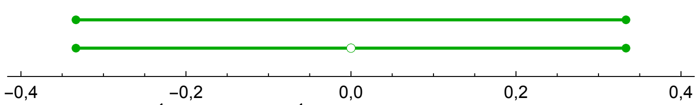
$-3x - 1 \leq 0$ et $3x - 1 \leq 0$

x		$-\frac{1}{3}$	
$-3x - 1$	+	0	−

x		$\frac{1}{3}$	
$3x - 1$	−	0	+

$([-\frac{1}{3}; \longrightarrow) \cap (\longleftarrow; \frac{1}{3}]) = [-\frac{1}{3}; \frac{1}{3}]$





dom $f = [-\frac{1}{3}; 0[\cup]0; \frac{1}{3}]$

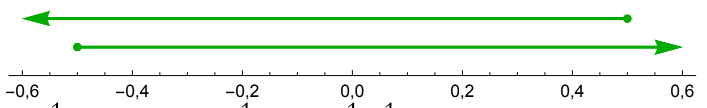
- 5) $f(x) = \dfrac{\text{Arccos}(2\,x)}{\text{Arctg}\left(\frac{x}{2}\right)}$

$\text{Arctg}\left(\dfrac{x}{2}\right) \neq 0 \Leftrightarrow x \neq 0 \Leftrightarrow x \in \mathbb{R} \setminus \{0\}$

$-1 \leq 2\,x \leq 1$

$-1 \leq 2\,x$ et $2\,x \leq 1$

$x \geq \dfrac{-1}{2}$ et $x \leq \dfrac{1}{2}$



$([-\frac{1}{2}; \longrightarrow) \cap (\longleftarrow; \frac{1}{2}]) = [-\frac{1}{2}; \frac{1}{2}]$

dom $f = [-\frac{1}{2}; 0[\cup]0; \frac{1}{2}]$

- 6) $f(x) = \text{Arcsin}\left(-x^2 - 2\,x + 1\right)$

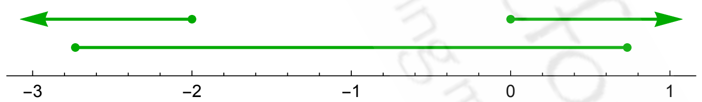
$-1 \leq -x^2 - 2\,x + 1 \leq 1$

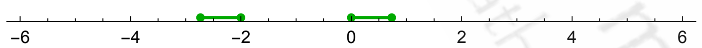
$-1 \leq -x^2 - 2\,x + 1$ et $-x^2 - 2\,x + 1 \leq 1$

$x^2 + 2\,x - 2 \leq 0$ et $-x^2 - 2\,x \leq 0$

x		$-1-\sqrt{3}$		$-1+\sqrt{3}$	
$x^2+2\,x-2$	$+$	0	$-$	0	$+$

x		-2		0	
$-x^2-2\,x$	$-$	0	$+$	0	$-$





$([-1-\sqrt{3}; -1+\sqrt{3}]) \cap (\longleftarrow; -2] \cup [0; \longrightarrow) = [-1-\sqrt{3}; -2] \cup [0; -1+\sqrt{3}]$

dom $f = [-1-\sqrt{3}; -2] \cup [0; -1+\sqrt{3}]$

- 7) $f(x) = \text{Arcsin}(2 - 5\,x)$

$-1 \leq 2 - 5\,x \leq 1$

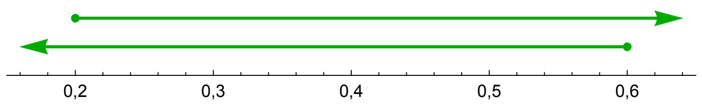
$-1 \leq 2 - 5\,x$ et $2 - 5\,x \leq 1$

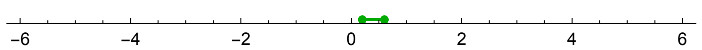
$5\,x - 3 \leq 0$ et $1 - 5\,x \leq 0$

x		$\frac{3}{5}$	
$5\,x-3$	$-$	0	$+$

x		$\frac{1}{5}$	
$1-5\,x$	$+$	0	$-$

$(\longleftarrow; \frac{3}{5}]) \cap ([\frac{1}{5}; \longrightarrow) = [\frac{1}{5}; \frac{3}{5}]$





dom $f = [\frac{1}{5}; \frac{3}{5}]$

- 8) $f(x) = \text{Arccos}\left(\dfrac{1}{x}\right)$

$x \neq 0$ et

$-1 \le \frac{1}{x} \le 1$

$-1 \le \frac{1}{x}$ et $\frac{1}{x} \le 1$

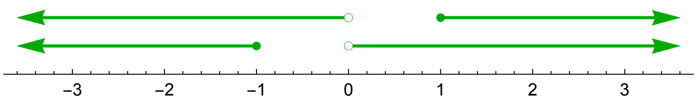
$-1 - \frac{1}{x} \le 0$ et $\frac{1}{x} - 1 \le 0$

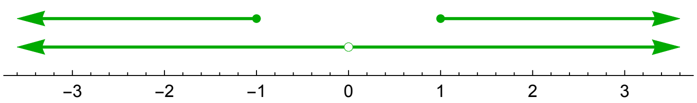
$\frac{-x-1}{x} \le 0$ et $\frac{1-x}{x} \le 0$

x		-1		0	
$\frac{-x-1}{x}$	$-$	0	$+$	||	$-$

x		0		1	
$\frac{1-x}{x}$	$-$	||	$+$	0	$-$

$(\longleftarrow; -1] \cup]0; \longrightarrow) \cap (\longleftarrow; 0[\cup [1; \longrightarrow) = \longleftarrow; -1] \cup [1; \longrightarrow$





dom $f = \longleftarrow; -1] \cup [1; \longrightarrow$

- 9) $f(x) = \text{Arcsin}(3\,x) - \text{Arccos}(2\,x)$

$-1 \le 2\,x \le 1$

$-1 \le 2\,x$ et $2\,x \le 1$

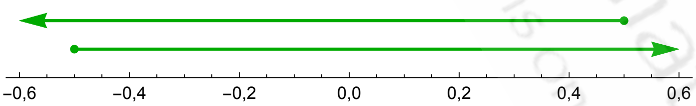
$-2\,x - 1 \le 0$ et $2\,x - 1 \le 0$

$-2\,x - 1 \le 0$ et $2\,x - 1 \le 0$

x		$-\frac{1}{2}$	
$-2\,x-1$	$+$	0	$-$

x		$\frac{1}{2}$	
$2\,x-1$	$-$	0	$+$

$([-\frac{1}{2}; \longrightarrow) \cap (\longleftarrow; \frac{1}{2}]) = [-\frac{1}{2}; \frac{1}{2}]$



$-1 \le 3\,x \le 1$

$-1 \le 3\,x$ et $3\,x \le 1$

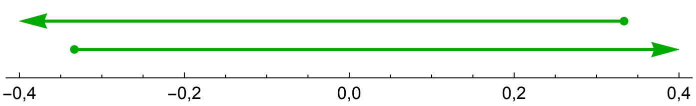
$-3\,x - 1 \le 0$ et $3\,x - 1 \le 0$

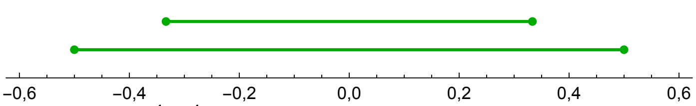
$-3\,x - 1 \le 0$ et $3\,x - 1 \le 0$

x		$-\frac{1}{3}$	
$-3\,x-1$	$+$	0	$-$

x		$\frac{1}{3}$	
$3\,x-1$	$-$	0	$+$

$([-\frac{1}{3}; \longrightarrow) \cap (\longleftarrow; \frac{1}{3}]) = [-\frac{1}{3}; \frac{1}{3}]$





dom $f = [-\frac{1}{3}; \frac{1}{3}]$

- 10) $f(x) = \text{Arctg}(6x^2 - 3x)$

dom $f = \mathbb{R}$

- 11) $f(x) = \text{Arccos}\left(\dfrac{x}{x+1}\right)$

$x + 1 \neq 0 \Leftrightarrow x \neq -1 \Leftrightarrow x \in \mathbb{R} \setminus \{-1\}$

$-1 \leq \dfrac{x}{x+1} \leq 1$

$-1 \leq \dfrac{x}{x+1}$ et $\dfrac{x}{x+1} \leq 1$

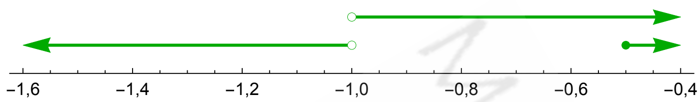
$-\dfrac{x}{x+1} - 1 \leq 0$ et $\dfrac{x}{x+1} - 1 \leq 0$

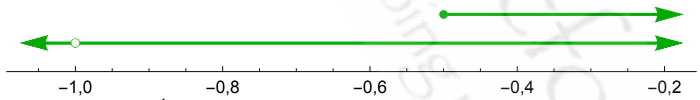
$\dfrac{-2x-1}{x+1} \leq 0$ et $-\dfrac{1}{x+1} \leq 0$

x		-1		$-\frac{1}{2}$	
$\frac{-2x-1}{x+1}$	$-$	|	$+$	0	$-$

x		-1	
$-\frac{1}{x+1}$	$+$	|	$-$

$(\longleftarrow; -1[\ \cup\ [-\frac{1}{2}; \longrightarrow) \cap (]-1; \longrightarrow) = [-\frac{1}{2}; \longrightarrow$

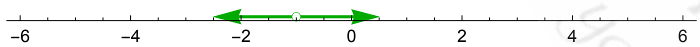




dom $f = [-\frac{1}{2}; \longrightarrow$

- 12) $f(x) = \text{Arctg}\,\dfrac{2x+1}{x+1}$

$x + 1 \neq 0 \Leftrightarrow x \neq -1 \Leftrightarrow x \in \mathbb{R} \setminus \{-1\}$



dom $f = \mathbb{R} \setminus \{-1\}$